USN

Third Semester B.E. Degree Examination, December 2011

Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. *Missing data will be suitably assumed.*

PART – A

1 a. Obtain the Fourier series for the function $f(x)=\begin{cases}\pi x & : 0 \le x \le 1 \\ \pi(2-x) & : 1 \le x \le 2\end{cases}$ and deduce that $\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. **(07 Marks)**

b. Obtain the half range Fourier sine series for the function. **(07 Marks)**
$f(x)=\begin{cases}1/4-x \ ; & 0 < x < 1/2 \\ x-3/4 \ ; & 1/2 < x < 1\end{cases}$.

c. Compute the constant term and the first two harmonics in the Fourier series of f(x) given by the following table. **(06 Marks)**

x :	0	1	2	3	4	5
f(x) :	4	8	15	7	6	2

2 a. Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & \text{for } |x|\le 1 \\ 0 & \text{for } |x|>1\end{cases}$ and hence evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}dx$. **(07 Marks)**

b. Find the Fourier cosine transform of $f(x)=\frac{1}{1+x^2}$. **(07 Marks)**

c. Solve the integral equation $\int_0^{\infty}f(\theta)\cos\alpha\theta d\theta=\begin{cases}1-\alpha \ ; & 0\le\alpha\le 1 \\ 0 \ ; & \alpha>1\end{cases}$. Hence evaluate $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$. **(06 Marks)**

3 a. Solve two dimensional Laplace equation $u_{xx} + u_{yy} = 0$, by the method of separation of variables. **(07 Marks)**

b. Solve the one dimensional heat equation $\frac{\partial u}{\partial t}=\frac{c^2\partial^2 u}{\partial x^2}$, $0< x < \pi$ under the conditions :
i) $u(0,+) = 0, u(\pi, t) = 0$ ii) $u(x, 0) = u_0\sin x$ where u_0 = constant ‡ 0. **(07 Marks)**

c. Obtain the D` Alembert's solution of one dimensional wave equation. **(06 Marks)**

4 a. Fit a curve of the form $y = ae^{bx}$ to the following data : **(07 Marks)**

x :	77	100	185	239	285
y :	2.4	3.4	7.0	11.1	19.6

b. Using graphical method solve the L.P.P minimize $z = 20x_1+10x_2$ subject to the constraints $x_1+2x_2 \le 40$; $3x_1+x_2 \ge 0$; $4x_1+3x_2 \ge 60$; $x_1 \ge 0$; $x_2 \ge 0$. **(06 Marks)**

c. Solve the following L.P.P maximize $z = 2x_1 + 3x_2 + x_3$, subject to the constraints $x_1 + 2x_2 + 5x_3 \le 19$, $3x_1 + x_2 + 4x_3 \le 25$, $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$ using simplex method. **(07 Marks)**

PART – B

5 a. Using the Regula – falsi method, find the root of the equation $xe^x = \cos x$ that lies between 0.4 and 0.6. Carry out four iterations. **(07 Marks)**

b. Using relaxation method solve the equations :
$10x - 2y - 3z = 205$; $-2x + 10y - 2z = 154$; $-2x - y + 10z = 120$. **(07 Marks)**

c. Using the Rayleigh's power method, find the dominant eigen value and the corresponding eigen vector of the matrix. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ starting with the initial vector $[1,1,1]^T$. **(06 Marks)**

6 a. From the following table, estimate the number of students who have obtained the marks between 40 and 45 : **(07 Marks)**

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	31	42	51	35	31

b. Using Lagrange's formula, find the interpolating polynomial that approximate the function described by the following table : **(07 Marks)**

x	0	1	2	5
f(x)	2	3	12	147

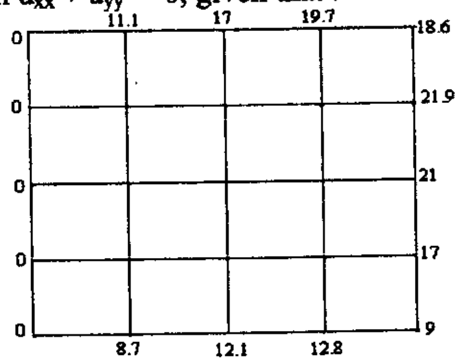
Hence find f(3)

c. A curve is drawn to pass through the points given by the following table :

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Using Weddle's rule, estimate the area bounded by the curve, the x – axis and the lines $x = 1$, $x = 4$. **(06 Marks)**

7 a. Solve the Laplace's equation $u_{xx} + u_{yy} = 0$, given that : **(07 Marks)**



b. Solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = 0$; $u(4, t) = 0$; $u(x, 0) = x(4 - x)$. Take $h = 1$, $k = 0.5$. **(07 Marks)**

c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x, 0) = \sin \pi x$, $0 \le x \le 1$; $u(0\ t) = u(1, t) = 0$ using Schmidt's method. Carry out computations for two levels, taking $h - 1/3$, $k = 1/36$. **(06 Marks)**

8 a. Find the Z – transform of : i) $(2n-1)^2$ ii) $\cos\left(\frac{n\pi}{2} + \pi/4\right)$ **(07 Marks)**

b. Obtain the inverse Z – transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. **(07 Marks)**

c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2n$ with $y_0 = y_1 = 0$ using Z transforms. **(06 Marks)**

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